B.Sc. part-I (Sub) Set Theory 03-04-2020.

Cartesian products.

Q.1 Show that

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Soln: Let $(x, y) \in A \times (B \cup C)$ be arbitrary. Then

$(x, y) \in A \times (B \cup C) \Rightarrow x \in A \wedge y \in (B \cup C)$

$\Rightarrow x \in A \wedge (y \in B \vee y \in C)$

$\Rightarrow (x \in A \wedge y \in B) \vee (x \in A \wedge y \in C)$

$\Rightarrow (x, y) \in A \times B \vee (x, y) \in A \times C$

$\Rightarrow (x, y) \in A \times B \cup A \times C$

$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ —— (1)

Again, let $(a, b) \in (A \times B) \cup (A \times C)$ be arbitrary. Then

$(a, b) \in (A \times B) \cup (A \times C)$

$\Rightarrow (a, b) \in (A \times B) \vee (a, b) \in (A \times C)$

$\Rightarrow (a \in A \wedge b \in B) \vee (a \in A \wedge b \in C)$

$\Rightarrow a \in A \wedge (b \in B \vee b \in C)$

$\Rightarrow a \in A \wedge b \in (B \cup C)$

$\Rightarrow (a, b) \in A \times (B \cup C)$

$\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ —— (2)

Now, from (1) and (2)

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Proved

Bishnupur part-I (Sub) Set Theory. 03-07-2020
Cartesian products.

Q. 2. Show that
$$A\times(B\cap C)=(A\times B)\cap(A\times C)$$

Soln: Let $(x,y)\in A\times(B\cap C)$ be arbitrary. Then

$(x,y)\in A\times(B\cap C)\Rightarrow x\in A\wedge y\in(B\cap C)$

$\Rightarrow x\in A\wedge(y\in B\wedge y\in C)$

$\Rightarrow (x\in A\wedge y\in B)\wedge(x\in A\wedge y\in C)$

$\Rightarrow (x,y)\in(A\times B)\wedge(x,y)\in(A\times C)$

$\Rightarrow (x,y)\in(A\times B)\cap(A\times C)$

$\therefore A\times(B\cap C)\subseteq(A\times B)\cap(A\times C)$ —— (1)

Again, let $(a,b)\in(A\times B)\cap(A\times C)$ be arbitrary.

Then, $(a,b)\in(A\times B)\cap(A\times C)$

$\Rightarrow (a,b)\in(A\times B)\wedge(a,b)\in(A\times C)$

$\Rightarrow (a\in A\wedge b\in B)\wedge(a\in A\wedge b\in C)$

$\Rightarrow a\in A\wedge(b\in B\wedge b\in C)$

$\Rightarrow a\in A\wedge b\in(B\cap C)$

$\Rightarrow (a,b)\in A\times(B\cap C)$

$\therefore (A\times B)\cap(A\times C)\subseteq A\times(B\cap C)$ —— (2)

Now, from (1) and (2), we get

$$A\times(B\cap C)=(A\times B)\cap(A\times C)$$

Proved.